Revista Mexicana de Economía y Finanzas Nueva Época
Volumen 14 Número 2, Abril-Junio 2019, pp. 203-219
DOI: https://doi.org/10.21919/remef.v14i2.307

Modeling and Projection of the Mexican Exchange Rate (Peso/Dollar): a Bayesian Approach for Model Selection

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(Primera recepción: 18/junio/2018, última recepción: 13/diciembre/2018, aceptado: 6/febrero/2019)

Abstract

This article studies the econometric modeling and the projection of growth rates of the nominal exchange rate (Peso/Dollar) from 1995 to 2018. Applying Bayesian simulation methods, the best data modeling fit between linear and non-linear econometric approaches is studied by introducing Markovian regime change parameters. The Bayes factor for model selection provides the following evidence: in the analysis of daily growth rates there are periods with low, medium, and high volatility. In the monthly rates, changes were also found in the mean and the volatility of the process. The linear autoregressive econometric model is not supported by the data in any case. Furthermore, instead of structural changes in these rates, evidence of state-dependent parameters is present. The high volatility in both data frequencies coincides with the sub-prime crisis in 2008-2009, but also with other sample periods. Moreover, an optimal weighting approach is applied to Markovian regime change models to study forecast errors in the sample. From this exercise, the forecasting errors of the exchange rate growth rates are lower than those of the linear autoregressive model. Finally, the out-of-sample errors of regime change models and optimal methods, in most cases, exceed those of linear inferences in both data frequencies.
JEL Classification: F31, C24, C11, G17
Keywords: Mexican exchange rate, Markov switching, Financial volatility, Bayesian analysis, Forecasting

Modelado y pronostico del tipo cambio de México (Peso / Dólar): Un enfoque Bayesiano para la selección del modelo

Resumen

Este artículo estudia el modelado econométrico y pronóstico de tasas de crecimiento del tipo de cambio nominal (Peso/Dólar) de 1995 a 2018. Aplicando métodos de simulación Bayesiana se estudia la mejor modelación de ajuste a los datos entre enfoques econométricos lineales y no-lineales introduciendo parámetros Markovianos de cambio de régimen. El factor de Bayes para seleccionar modelos proporciona la siguiente evidencia: en el análisis de tasas de crecimiento diarias hay periodos con baja, media y alta volatilidad. En las tasas mensuales, también se encontraron cambios en la media y la volatilidad del proceso. El modelo econométrico autorregresivo lineal no es soportado por los datos en ningún caso. Además, en lugar de los cambios estructurales en dichas tasas, hay evidencia de parámetros dependientes del estado. La alta volatilidad en ambas frecuencias de datos coincide con la crisis *sub-prime* en 2008-2009, pero también con otros períodos de la muestra. Mas aún, se aplica un enfoque de ponderación óptimo a modelos Markovianos de cambio de régimen para estudiar los errores de pronóstico en la muestra. De este ejer-

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*Sin fuente de financiamiento declarada para el desarrollo de la investigación

Resumen

cicio, los errores de pronóstico de las tasas de crecimiento del tipo de cambio son menores a los del modelo lineal autorregresivo. Finalmente, los errores fuera de la muestra de modelos de cambio de régimen y métodos óptimos, en la mayor parte de los casos, superan aquellos de las inferencias lineales en ambas frecuencias de los datos.
Clasificación JEL: F31, C24, C11, G17
Palabras clave: Tipo de cambio en México, Parámetros Makovianos, Volatilidad Financiera, Análisis Bayesiano, Pronóstico

1. Introduction

The study exchange rate in emerging markets is a subject of great importance for academics, financial researchers, central bank and decision makers, where the concept of volatility can be approximated as a measure of risk. It is usually measured by deviations of the variance of the studied series. However, different types of risk exist and there is no consensus since new empirical models emerge when new data arises (Granger 2002). For example, it is argued that a linear statistical model cannot capture cyclical patterns observed in financial and economic time series (Hamilton 2005).

There are studies of exchange rate markets about its determinants, efficiency hypothesis, forecasting and volatility. Some of them apply linear autoregressive models (AR), conditional heteroskedasticity modeling (GARCH) and Markov Switching (MS) specifications. However, the appropriate adjusting of the model to the data is not always discussed. That is, in most of the literature, the model specification is exogenously imposed by assumption. Therefore, the economic implications and forecasting might change under a misspecification of the model. For example, in Frühwirth-Schnatter (2001) is discussed the number of states assumed in the switching specification of Engel and Kim (1999), who studied the U.S./U.K. real exchange rate.

The literature in switching modeling is huge. The likelihood of persistence or changes in the mean and volatility of the process can be modeled using the regime switching model, that allows state dependent parameters over time. From the seminal works of Hamilton (1989), Hamilton and Susmel (1994), and in particular the paper of Engel (1994) about exchange rates, there is a plenty of research in macroeconomics and financial econometrics with regime switching modeling (e.g. co-movements in economic variables; changes in volatility in cycle fluctuations; switching DSGE models; business cycle turning points in real time and forecasting; and many more), see the surveys of Diebold and Rudebusch (1999), Kim and Nelson (1999b), Hamilton and Raj (2002) and Frühwirth-Schnatter (2006). However, the MS framework has been assumed in many occasions without testing its existence.

Studies of the foreign exchange rate markets with MS frameworks attempt to answer different financial issues, Clarida et al. (2003), Chen (2010), Psaradakis et al. (2004) among others. For the Mexican exchange rate, Brown and Curci (2002) studied the relationship between the volatility in the Mexican peso spot market and futures contracts trading activity by a linear auto-regression vector (VAR); a MS autoregressive model with two states to explain the bias implied in the peso forward market is suggested in Bazdresch and Werner (2005); and Benavides and Capistrn (2012) mixed GARCH models by using switching indicators to change between different volatility specifications. They claimed that these combinations of models were statistically superior in terms of forecasting performance to individual models.

The present paper is very close to Ibarra Salazar et al. (2017) and Islas-Camargo et al. (2017). In the first, determinants of the nominal exchange rate by structural models suggested by the financial theory were studied. They imposed and proved the existence of a structural break in the sub-prime crisis in 2008 to 2009, but this break should be endogenous to the model. In the second work, it is argued that a two state MS model to study the hypothesis of efficiency in the forward exchange rate leads to different economic implications than a linear model. However, the model selection based on the likelihood ratio test is subject to the problem of nuisance parameters (Carrasco et al. 2014). Our study complements the findings of these two works.

Hence, the contribution of the present paper is to study the econometric modeling and forecasting of the nominal Mexican exchange rate (Peso/Dollar). Under a Bayesian approach, the model selection of the econometric modeling for the exchange growth rates is carried out. This selection suggested is based on the Bayes factor which tests the number of lags, parameters subject to regime switching, and the number of states. The model specification issue should not be discarded since the economic implications, forecasting and conclusions are based on the most reliable model conditional to the data. The associated strong financial shocks of the Mexican exchange market yield that the assumptions of the standard linear econometric models might not hold. This paper does not study determinants to explain volatility exchange rate, financial issues of this market, nor an explanation of the market shocks. Instead, it provides empirical evidence for an appropriate modeling and forecasting of this exchange market given by their own dynamics to study similar topics in future research.

The presence of MS parameters in an observed single equation and model selection is studied in Frühwirth-Schnatter (2004) and Carrasco et al. (2014). These works provide two different approaches to test for that. The first is based on Bayesian methods and the latter is based on testing the parameters on the linear modeling under the null hypothesis by the frequentist approach. On the other hand, the model selection in high frequency data and GARCH modeling with MS frameworks is studied by the Bayesian approach in Bauwens et al. (2014). They claim that, if there is evidence of structural breaks or state dependence in the volatility, the MS frameworks are preferred over the standard GARCH models. However, in our paper we only focus on autoregressive modeling with state dependent volatility in a wide set of Markov switching heterogeneity. It can be shown that, this is nested in a GARCH model with MS parameters, and the periods of time inferred with changes in the states are very similar.

The empirical financial time series in emerging markets are characterized by high instability Aggarwal, Inclan, and Leal (1999), the stylized facts about these series are usually very volatile and have excess kurtosis, asymmetries, non-normal distributions and absence of linear autocorrelation (Fama 1965), volatility clusters (Cont 2001), and non-constant volatilities, see Loschi et al. (2005). Therefore, an econometric model for the Mexican exchange rate with constant parameters over time and the assumptions related to linear modeling might not be very useful. For example, in Mexican financial data Lopez Herrera (2004), Lopez-Herrera and Venegas-Martinez (2011), Lopez-Herrera et al. (2011) and Heath and Kopchak (2015) have studied Markovian models and volatility, but in these studies the model specification has been imposed by the researchers. An exception is the work of Cabrera et al. (2018) where changes in the volatility of the Mexico stock exchange are studied. Based on the Bayes factor model selection there were found three different states, and the growth rates of the stock exchange in the sub-prime crisis are captured in the high volatility state. However, any forecasting exercise was neglected in the last paper. Our paper is very close to the later, but the Markov switching heterogeneity has been extended, and a forecasting exercise is carried out.

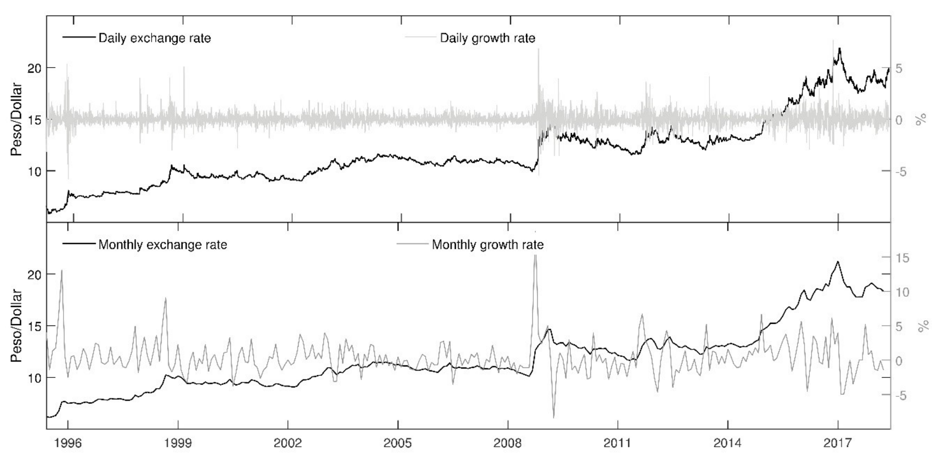
Finally, Markov switching models have been recognized from a discrepancy between in-sample and out-of-sample performance. In-sample analysis the Markov switching models suggest interesting features like changes of the financial and economic time series given by cycle fluctuations and volatility. However, the out of-sample performance, is frequently inferior to simple linear models in terms of loss functions. This issue was studied in Boot and Pick (2017). They provided an optimal weights in Markov switching frameworks that can beat the linear models forecasting errors by choosing the model specification that minimize the mean of these errors. Different to this work, we proceed as follows: by applying Bayesian simulation methods, the estimation of the Marginal Likelihood (ML) is carried out similar to Frühwirth-Schnatter (2004), but for a wider set of MS heterogeneity models. This, to provide enough evidence of best fit to data model (Mexican exchange growth rates from 1995 to 2018 in daily and monthly frequency). The model selection is based on the Bayes factor according to Kass and Raftery (1995). Once the best fit to data model is chosen, there will be discussed some important issues about economic implications, volatility and forecasting of the Mexican exchange growth rates and levels. The last, based in the optimal weighting approach in MS models of Boot and Pick (2017).

The structure of the paper is as follows: in Section 2 there are reported some statistical properties of the Mexican exchange rate (Peso/Dollar); in Section 3 the econometric modeling and the Bayesian estimation procedure are described. Moreover, some issues about model selection are discussed; in Section 4 the empirical application results, the exchange rate in-sample and out-of-sample forecasting are discussed. Finally, conclusions are presented in Section 5.

2. Nominal Mexican exchange rate (Peso/Dollar)

In this section, the statistical moments and plots of the nominal exchange rate (Peso/Dollar) are discussed. The data is studied in exchange growth rates at different frequencies (i.e., daily and monthly data) from April 1995 to May 2018. The analysis suggests that independent of the frequency, the nominal exchange rates are not stationary time series, and the sample moments present changes over time.

Figure 1. Mexican exchange rate (Peso/Dollar)



Source of data: Central Bank of Mexico

In Figure 1, the daily and monthly exchange rate and their percentage of growth are plotted together to compare the data at different frequencies. These plots show several periods of high volatility beside the sub-prime crisis from 2008 to 2009. Therefore, the graphical evidence suggests that the volatility might not be constant over time. From both plots, the nominal exchange rates are not stationary time series. It is omitted the standard stationary tests at the literature, since they are based on constant parameters over time. This last assumption does not hold as it will be shown later. However, Ibarra

Salazar et al. (2017) presented these tests that reject the hypothesis of stationary monthly exchange rate.

In Table 1, it is presented the sample mean and the standard deviation (Std) of the daily and monthly exchange growth rates by year and for the full sample. The mean of the daily data is very close to zero in every year and 0.02 for the full sample. This means that the daily average growth rate is positive, so the exchange rates increase over time. The associated daily volatility, given by the standard deviation, is close and greater than one in the years 1995, 2008, 2009 and 2016. That is, in these years (including the sub-prime crisis) there are high daily volatility of the exchange rates. The average of the daily volatility in the full sample is 0.64. On the other hand, the annual mean of the monthly growth rate is higher in the years 1995, 2008, 2015 and 2016, while the monthly growth rate average mean of the full sample is 0.50. Therefore, the monthly average growth of the exchange rates is positive and high. The years of monthly high volatility are 1995, 2008, 2009, 2016 and 2017, and the average monthly volatility is 2.32. In summary, there is evidence that the volatility and the mean of the exchange growth rates change over time.

Table 1. Sample moments of the Mexican exchange growth rates

Data	**Year**	**1995**	**1996**	**1997**	**1998**	**1999**	**2000**	**2001**	**2002**	**2003**
Daily	Mean	0.08	0.01	0.01	0.08	-0.01	0.01	-0.02	0.05	0.03
	Std	1.31	0.44	0.56	0.7	0.6	0.48	0.49	0.49	0.57
Monthly	Mean	4.25	0.32	0.4	1.61	-0.48	0.11	-0.24	0.85	0.77
	Std	5.12	1.51	1.7	3.26	1.78	1.89	1.63	1.41	2.31
	Year	**2004**	**2005**	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**	**2012**
Daily	Mean	0	-0.02	0.01	0	0.1	-0.02	-0.02	0.05	-0.03
	Std	0.4	0.37	0.45	0.35	1.16	0.9	0.63	0.76	0.66
Monthly	Mean	0.18	-0.53	0.21	-0.02	1.7	0.06	-0.49	0.91	-0.36
	Std	1.45	0.9	1.85	1.15	5.79	3.54	1.94	2.79	2.71
	Year	**2013**	**2014**	**2015**	**2016**	**2017**	**2018**	**Full sample**		
Daily	Mean	0.01	0.05	0.06	0.08	-0.02	0	Mean		0.02
	Std	0.65	0.4	0.66	1.02	0.65	0.7	Std		0.64
Monthly	Mean	0.02	0.35	1.7	1.59	-0.43	-0.37	Mean		0.5
	Std	2.05	1.01	2.12	3.12	3.26	1.33	Std		2.32

3. Econometric modeling

The notation of this section is based in Frühwirth-Schnatter (2004) and Timmermann (2000). Let x_t be the exchange rate (Peso/Dollar) and the stationary time series $y_t = 100\,(x_t - x_{t-1})\,/x_{t-1}$ of daily/monthly growth rates. To model the dynamic of the growth rates for both data frequency, the following autoregressive models are suggested: First, a Markov Switching autoregressive model (MSAR), where the full set of parameters is state dependent, given by:

$$\phi_{S_t}(L)\, y_t = \zeta_{S_t} + \sigma_{S_t}\varepsilon_t \qquad (1)$$

Second, the specification suggested by Hamilton (1989) where the role of the hidden indicators are more involved since the difference respect to the mean of the process is switching (MSMAR):

$$\Phi(L)\left(y_t - \mu_{S_t}\right) = \sigma_{S_t}\varepsilon_t \qquad (2)$$

Third, the linear autoregressive model (LAR):

$$\phi(L)\, y_t = \zeta + \sigma\varepsilon_t \qquad (3)$$

where $\varepsilon_t\,(0,1)\,;\Phi\,(L)$ is a polynomial of order p and L is the lag operator of the autoregressive parameters which are state dependent in model (1); the switching intercept ζ_{S_t} which leads to a state dependent mean in model (1); the switching mean $\mu_{S_t} inmodel(2), and\sigma_{S_t}$ captures the state dependent volatility due to the error term. The hidden indicator S_t of the states is assumed to follow a discrete order one Markov process, i.e., the transition probability from state k to state l is given by $\xi_{kl} = \Pr\left(S_t = l_{t-1} = k\right)$ for all $t = 1, ..., T$ and $k, l \;\in \{1, \ldots, K\}$ states. Let's denote the augmented parameter set as $\Psi = (\psi, S)$, wich includes $S = (S_1, \ldots, S_T)$ and $\psi = (\phi_k, \zeta_k, \mu_k, \sigma_k, \xi_{kl})$, where the sub index k means the state dependent parameter.

It is important to note that, if $K = 1$ model (1) and (2) collapse to model (3). However, if $K > 1$ MSAR and MSMAR are not nested models. They have different moments, dynamics, and statistical properties. See Timmerman (2000) for a proof. Moreover is important to discuss the state-paths dynamics among model (1) and model (2). In the first, conditional on the hidden indicators, the likelihood leads to Kdifferent paths at every time period; and for the second, the paths are (K^{1+p}) where p is the number of lags in the polynomial operator. Therefore, in both models the likelihood for all t is given by t to the power of the number of paths, which is not operable even in short time periods. The dependency of the hidden indicators on the states allows to capture long memory in the process as documented in Diebold and Inoue (2001). In particular, the model (2) leads to richer dynamics in short term since the lags of the state dependent mean, and this captures better an approximation of the long memory in the process.

To make operable the likelihood, approximations have been suggested at the literature: Hamilton (1989), Kim and Nelson (1998) and a particle filter in Bauwens et al (2014). They are based on the stationary ergodic properties of the Markov process of the hidden indicators. In this paper, for model (1) these approximations to estimate the marginal likelihood of the model are applied. Furthermore, the MSMAR model was not studied in Frühwirth-Schnatter (2004) since the problem of state dependency paths even in low finite memory of the hidden indicators. I extend their approach to account for that by applying the Hamilton's filter approximation.

The mixture distributions given by Markov switching models are able to generate probability distributions with asymmetry and fat tails. Timmermann (2000) demonstrated that the introduction of Markovian dependence into the hidden indicators expands the scope for asymmetry and fat tails that can be generated by mixture modeling. This explains the relative success that these models have had in applications to financial and economic time series. The Markov switching models lead to an interesting feature that generates processes with y_t and y_t^2 being autocorrelated. In particular, from Proposition 2 in Timmermann (2000), even though the process y_t^2 is uncorrelated conditional on knowing the state, autocorrelation in y_t^2 enters through persistence of the states, and the marginal distribution has fat tails, as long as far away are the state dependent variances. Moreover, if high volatility states are followed by states with either high volatility or a mean parameter far away from the unconditional mean, this will tend to create fat tails and increase kurtosis.

In summary, there are $K(2p + 1)$ possible autoregressive specifications[2] to model the exchange growth rates (Peso/Dollar) in daily and monthly frequency. Thus, this paper attempts to find the best fit model conditional to data. For this, simulation Bayesian methods based on Frühwirth-Schnatter (2004) are applied and extended to account for the MSMAR model as mentioned before, see the next section about the estimation procedure. By the bridge sampling technique (Meng and Wong 1996), and the Markov Chain Monte Carlo (MCMC) simulation methods, is possible to estimate the marginal likelihood (ML)

[2]The maximum number of lags considered is two since the number of paths in the MSMAR increases exponentially with the number of lags (K^{1+p}). In previous version of the paper, eight lags in the model (1) were considered, but the Bayes factor provided evidence to only two lags as the best fit to data model.

for each model i, i.e., $\Pr(y_i)$. Therefore, the Bayes factor allows to compare the model j with the highest marginal likelihood against of model i as follows:

$$BF_{i,j} = \frac{\Pr(y_j)}{\Pr(y_i)} = \frac{\Pr(M_j)}{\Pr(M_i)}$$

where $\Pr(y_j)$ is the ML of data y conditioned to model j, and $\Pr(M_j)$ is the probability of model j conditioned to data y. Note that there is no prior probability assigned to any model, i.e., $\Pr(M_j) = \Pr(M_i)$. Therefore, it is possible to infer the model probability conditional on data from the ratio of ML. In Table 2, the guide of model selection suggested by Kass and Raftery (1995) is reported. Thus, the probabilities of the data conditioned to model i and j for different values of the for the Bayes factor in log_{10} can be obtained. Let the model j leads to the highest marginal likelihood, if $log_{10}(BF_{i,j}) > 3$, there is positive evidence against of model i and this specification can be rejected. On the other hand, if the Bayes Factor is less or equal than three, both models represent the data with high probability. Therefore, we should be indifferent between choose one of them.

Table 2. Bayes factor and model selection

Bayes factor (log_{10})	**Evidence against of the model i**
1 a 3	Not worth more than a bare mention
3 a 20	Positive
20 a 150	Strong
Más de 150	Very strong

Guide of model selection through the Bayes factor of Kass and Raftery (1995)

It is important to mention, that the ARIMA models are mainly focused on the short-run properties of the data misleading the of long-run properties. The Autoregressive Fractionally Integrated Moving Average (ARFIMA) models provide an alternative to ARIMA models allowing to exhibit stationary ARMA behavior after being fractionally differenced, Sowell (1992). However, Diebold and Inoue (2001) provide a theoretical explanation, and their simulation study demonstrate that when structural change or stochastic regime switching exists, they are related to long memory and are easily confused with it. Furthermore, they argue that long memory and regime switching are interchangeable concepts. In our paper, the ARFIMA modeling in Markov switching frameworks is omitted. However, this might be studied as follows: The parameter of the fractional difference operator can be treated as a random variable in a continuous finite space. It can be included in the Bayesian MCMC scheme of the next section by generating draws from a truncated density in an additional MCMC block sampler step. Alternatively, the dynamics of this parameter can be treated as a time varying parameter in a switching state space model. However, this issue and the Bayesian simulation methods of model comparison are left for future research.

3.1 Bayesian estimation procedure

The simulation of the ML to each autoregressive model is carried out based on Frühwirth-Schnatter (2004), further details and the Matlab code to estimate the marginal likelihood of the model (1) and (3) can be found in the work of Frühwirth-Schnatter (2006). To estimate the marginal likelihood of model (2) when the hidden indicators of the states are more involved, I proceed in a similar fashion as Frühwirth-Schnatter (2004), but applying the Hamilton's filter and the collapsing technique of Kim and Nelson (1998) to reduce the number of paths to simulate the marginal likelihood. Given the ML estimation, the Bayes Factor for every possible combination of models can be obtained by the log_{10} of the ratio. Finally, once the model has been chosen, a new restricted MCMC is carried out for the best fit to data model-specification. Restricted sampling Monte Carlo means that the identification restrictions for the state dependent parameters are imposed. That is, if the preferred autoregressive specification is a 3-state model, a possible restrictions of the MCMC sampling scheme are: $\sigma_1^2 > \sigma_2^2 > \sigma_3^2$ and $\mu_1 < \mu_2 < \mu_3$. The procedure of the restricted sampling MCMC is as follows:

- For initial values of the hidden indicators of the states S and variances for the error term, sampling the parameters ψ conditional to data and S from the $p(\psi, S)$. The autoregressive parameters are sampled from a multivariate normal, the variance of an inverted gamma, and the transition probabilities from a Dirichlet probability density function.
- Apply the accepted rejected sampling to the last step by imposing the state dependent identification restrictions. That is, if the sampled parameters satisfy the restrictions save the sampled draws, otherwise keep with the previous one.
- Given the parameters from the last step, sampling the hidden indicators of the states from the forward-backward-smoothing algorithm described in Frühwirth-Schnatter (2006).
- Repeat all the steps with the draws as the initial values, 7,000 times discarding the first 2,000 to eliminate the dependency of the initial values.

Given the restricted MCMC output, all the posterior sample moments for the parameters can be computed (e.g., the sample mean and standard deviation). Moreover, from the posterior mean, the hidden indicators of the states can be simulated, and the associated probabilities to every state for $t = 1, ..., T$ are called the filtered and smoothed probabilities in the literature.

3.2 Issues in model selection

Testing the linear model versus a MS autoregressive model cannot be carried out by the likelihood ratio test because, under the null hypothesis that the linear model is true, the MS parameters are not identified. This is called the problem of nuisance parameters at the literature, see Andrews and Ploberger (1994), Hansen (1996) among others. Furthermore, the ratio test does not have an asymptotic chi-square distribution since the information matrix is singular. An optimal test that attempts to solve these issues is suggested in Carrasco et al. (2014). This test only requires estimating the model under the null hypothesis where the parameters are constant. However, this is not useful for testing among different MS heterogeneity (e.g., a two versus three states model, a MSAR model (1) against of MSMAR specification given by (2). Moreover, these tests are based on asymptotic properties. That is, they are only valid if there are enough number of observations in each state, but the likelihood regular optimal conditions could fail when there are few observations in some state (Frühwirth-Schnatter 2006).

From the last discussion, it is preferred to carry out a model selection based on the Bayesian approach. From this, the Bayes factor can always be estimated by simulation methods even if the MS parameters are not identified, and the number of observations is not large enough in some of the states. As it was mentioned, this model selection is based on simulate the ML for each model applying an unrestricted MCMC sampling scheme (that is, without imposing any prior parameter identification restriction). See Frühwirth-Schnatter (2004) and Bauwens et al. (2014) for the main references in univariate MS autoregressive and MS-GARCH models respectively.

4. Results

In this section, the monthly and daily exchange growth rates (Peso/Dollar) from April 1995 to May 2018 are studied. The source of the data is the Central Bank of Mexico. First, the model selection provides evidence of MS frameworks in both data frequency. Second, the estimation and inference of the models with best fit to data are reported. Finally, in both frequency data, the forecasting exercise leads to lower prediction errors in-sample and out-of-sample inferences in MS modeling than those from linear specifications.

4.1 Model selection

In Table 3, for the monthly exchange growth rates (Peso/Dollar) the estimation of the ML based on the simulation Bayesian methods described in the last section, and the Bayes factor are reported. The Bayes factor was estimated related to the model with the highest ML marked in bold. The number of lags considered were one and two, see the footnote (1); and the number of states goes from one, which leads to the LAR model of equation (3), to two and three for the MSAR and MSMAR models, equations (1) and (2) respectively.

Table 3. Marginal likelihood and Bayes factor (monthly data)

	LAR	**MSAR**	**MSMAR**	**MSAR**	**MSMAR**
Lags	K=1	K=2		K=3	
1	-649.6833	-618.6278	**-618.1775**	-619.0034	-618.9273
2	-649.9278	-619.7635	-619.1190	-620.2166	-619.1119
1	13.6828	0.1956	0.0000	0.3587	0.3257
2	13.7890	0.6888	0.4089	0.8856	0.4058

The first rows report the estimation of the ML by the bridge sampling technique. The last part of the table presents the Bayes factor related to the model with highest ML (marked in bold).

According to the guide of Kass and Raftery in Table 2, there is positive evidence against of the linear models, thus they can be rejected. However, two and three states can represent the monthly growth rates data with high probability. This, because all the Bayes factor related to the highest ML (marked in bold) are less than three. Even though, the model with the highest ML is the specification given by MSMAR(1) with two states, a model with two lags is carried out to study the monthly data and the forecasting exercise. This, since both models have high support by the data, and to compare the results with the daily data where three states are preferred according with Table 4.

In the next table, the estimation of the ML by the bridge sampling technique and the Bayes factor of daily exchange growth rates are reported. From the guide of the Bayes factor, the MSAR model with three states is preferred. Evidence very strong against linear models are provided. Moreover, strong evidence against of two states models, and positive evidence against MSMAR specifications with three states were found. Finally, the three states MSAR models with one and two lags have high probability to represent the daily data. Therefore, two lags are considered to study the inference and forecasting of the daily exchange growth rates.

Table 4. Marginal likelihood and Bayes factor (daily data)

	LAR	**MSAR**	**MSMAR**	**MSAR**	**MSMAR**
Lags	K=1	K=2		K=3	
1	-5968.9631	-4919.7918	-4917.0458	**-4713.6011**	-4731.9395
2	-5970.8091	-4924.0953	-4919.0491	-4719.1882	-4732.2134
1	>150	89.5475	88.3549	0.0000	7.9643
2	>150	91.4165	89.2249	2.4264	8.0832

The first rows report the estimation of the ML by the bridge sampling technique. The last part of the table, presents the Bayes factor related to the model with highest ML (marked in bold).

4.2 Estimation and inference

Once the best models are chosen, the restricted MCMC algorithm described in the last section is carried out to estimate the models for different frequency data. In Table 5 the posterior moments of the process (sample mean and standard deviation as a measure of volatility) are presented[3]. That is, for each state, $S_t = 1, .., K$, the posterior mean for the mean and the volatility of the processes are estimated. The High Posterior Density Interval (HPDI) at the 95 % is also reported in this table. This interval is the posterior draw for each parameter in the 2.5 % and 97.5 % position. This is equivalent to the percentiles in frequentist econometrics.

In this table, for the daily data, the states one and two have means close to zero (-0.0146 and 0.0456) with associated volatilities of 0.3651 and 0.7333. However, all of these moments are enough different, see the HPDI. Therefore, these states are denoted as low and medium volatility regimens. Furthermore, in the third state, the mean is higher than the others (0.3790) characterized with the highest volatility (2.3305), but the persistence and the probability for each state are different as it will be shown later. On the other hand, in the monthly data, there are two different means of the process: the first growth rate mean (1.2681) and the second zero (that is, the HPDI at the 95 % does not exclude zero in the first state). Finally, the volatility of the non-zero growth exchange rate is 4.5538 which is higher than the volatility of zero monthly mean (1.6077).

Table 5. Posterior moments form the MCMC output

Daily exchange growth rates					**Monthly exchange growth rates**				
State	**Parameter**	**Mean**	**HPDI**		**State**	**Parameter**	**Mean**	**HPDI**	
1	Mean	-0.0146	-0.0285	-0.0025	1	Mean	0.0003	-0.4019	0.3885
	Volatility	0.3651	0.3508	0.3789		Volatility	1.6077	1.3007	1.9142
2	Mean	0.0456	0.0124	0.0798	2	Mean	1.2681	0.3385	2.3372
	Volatility	0.7333	0.6949	0.7709		Volatility	4.5538	3.4707	6.3206
3	Mean	0.3790	0.0957	0.7176					
	Volatility	2.3305	2.0171	2.7236					

Mean and volatility of the process were estimated from the restricted MCMC output from the models: MSAR and MSMAR for daily and monthly data respectively. Both specifications with two autoregressive lags.

In Table 6, the posterior mean of the transition matrices and the steady state probabilities are reported. The transition probability parameters provide important information about the state persistence, which is the transition probability that to be in state k in the current time-period and stay in the same state in the next period. Furthermore, from the transition matrix it is possible to estimate the marginal probability of each state. They are called the steady state probabilities at the literature.

The persistence of the daily data is as follows: the probability to be in high volatility and stay in the same state is 0.8515, the persistence of medium and low volatility are 0.9588 and 0.9786 respectively. This means that the highest persistence is in the low volatility state. The percentage of times that this low volatility is present is around the 57 % of the sample. However, only 3 % of the time the process is in the high volatility state, and close to 40 % in medium volatility. In monthly data, the persistence of the high and low volatility are 0.6557 and 0.8964 respectively. They are not so high, which means it is likely that the states constantly change every month. The percentage of times that the high volatility with growth rate are present is around 23 %, and close to 77 % of the time in low volatility with zero mean.

[3] All the posterior moments of equations (1) and (2) were omitted to save space.

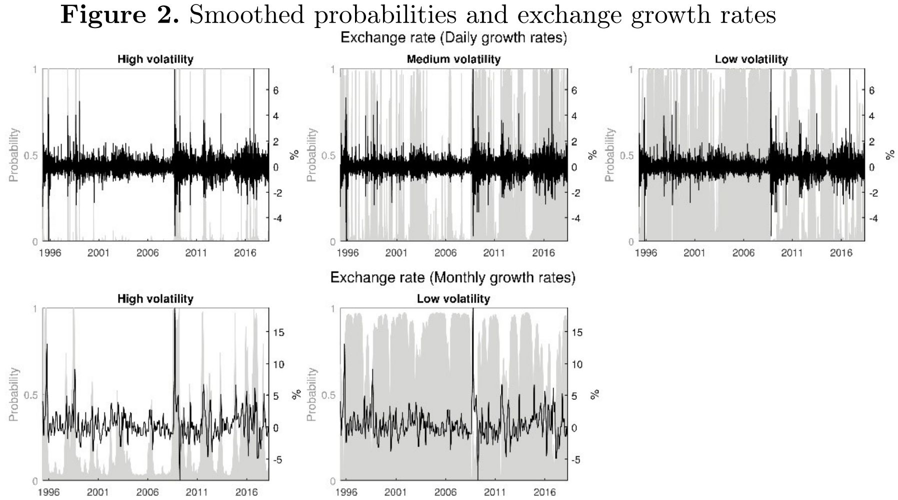
Table 6. Transition matrix and steady state probabilities

Daily exchange growth rates					Monthly exchange growth rates			
TM	$S_t = 1$	$S_t = 2$	$S_t = 3$	**SSP**	**TM**	$S_t = 1$	$S_t = 2$	**SSP**
$S_t = 1$	0.9786	0.0209	0.0005	0.5702	$S_t = 1$	0.8964	0.1036	0.7686
$S_t = 2$	0.0302	0.9588	0.0109	0.3986	$S_t = 2$	0.3443	0.6557	0.2314
$S_t = 3$	0.0043	0.1442	0.8515	0.0312				

Transition matrix (TP) and steady state probabilities (SSP) from the restricted MCMC output

In the Figure 2 the smoothed probabilities for $t = 1, ..., T$ are plotted with the growth rates of both data frequency. These probabilities sum one for all t. Therefore, if the smoothed probabilities tend to one in the state k the exchange growth rate moves to this state. If probabilities decreases a change of the state is likely. In the top plots of this figure, the gray area corresponds to the smoothed probabilities for the three states of daily data. In the bottom plots, there are presented the two states of monthly data with their associated smoothed probabilities.

Figure 2. Smoothed probabilities and exchange growth rates



The smoothed probabilities were estimated by the posterior mean from the restricted MCMC output, by the forward-backward-smoothing algorithm based on Frühwirth-Schnatter (2006).

The periods of high volatility of daily growth rates are well captured by the gray area of the smoothed probabilities. There are several days of high volatility in the years of 1995, 2008, 2009, 2016 and 2017 and some other dates, which are the most volatile. This includes the sub-prime crisis times. The periods of medium and low volatility represented by the gray area are approximated to 39.86 % and 57.02 % respectively. They capture in an appropriate way the changes in the volatility. Notice that, from 2016 up to now the probability to be in any state is higher than in the rest of the sample, which leads to a market highly volatile with low persistence during the last two years.

In the case of monthly data, the area of high volatility is the 23.14 %. The sub-prime times and some others are captured by these smoothed probabilities. However, the area on the low probability (76.86 %) is greater but does not reach to one at any time. This means that, each month there is a probability that the low volatility goes to high. However, the high volatility state is present with high probability in the last two years. These bottom plots show the low persistence of the states as mentioned before from the transition matrix parameters of Table 6.

The smoothed probabilities plots are one of the most important features of MS modeling. There is always a probability to change to another state, then there is not a structural change. In models with structural breaks, it would not be possible to change to the any previous state again. This is a strong assumption which in general might not hold. Even more, the linear modeling assumes that there is only one state in the full sample (unless exogenous structural breaks are imposed).

The model specification in monthly data based on the Bayes factor criteria, coincides with Islas-Camargo et al. (2017) supporting a MS specification over the linear autoregressive model, who studied the efficiency in the forward exchange rate. However, in high frequency exchange rate (Peso/Dollar) data, the MS specification suggests three states instead of two. In that paper, it is argued that the likelihood ratio test leads to a MS framework with two states, but this test is not valid as it was mentioned before. Therefore, the number of states might be misspecified. Even though the sample of that work covered 2002 to the beginning of 2017, a three states model was supported by the data according with our results and Figure 2.

Finally, in Table 7 the p-value in frequentist econometrics tests on the residuals are reported. These residuals were estimated from the posterior mean of the MCMC output to the full time series (All), and conditional to every state for each frequency of the data. That is, two states in monthly data and three for daily data. In monthly data, all the null hypothesis cannot be rejected, therefore residuals are not correlated, and no ARCH effects are present, which means that the MSMAR(2) with two states leads to white noise error term in each state and for the whole sample. However, in daily data the null hypothesis are rejected for the whole sample (All). But, conditional to every state, the MSAR(2) with two states leads to white noise error terms.

Table 7. Residual test (p-value)

	Monthly residuals			**Daily residuals**			
Test	**All**	**S_t=1**	**S_t=2**	**All**	**S_t=1**	**S_t=2**	**S_t=3**
Ljung-Box Q-test	0.3591	0.0564	0.9453	0.0000	0.9739	0.8770	0.7924
Ljung-Box Q-test	0.8060	0.5851	0.9771	0.0000	0.1022	0.4572	0.0020
Engle's ARCH test	0.3731	0.5712	0.5134	0.0000	0.3547	0.5860	0.0678

Null hypothesis: The residuals are not autocorrelated, the square residuals are not autocorrelated, and no conditional heteroscedasticity in this order respectively.

4.3 Exchange rate (Peso/Dollar) forecasting

There is a wide literature of forecasting in economic time series and the exchange rate data in MS frameworks from the works of Hamilton and Susmel (1994) and Engel (1994) respectively. However, the approach of Boot and Pick (2017) is applied in the present paper to forecasting the Mexican exchange rate (Peso/Dollar). This approach suggests forecasting in MS frameworks based on optimal weights for the full sample. Thus, the mean of the square forecasting error (MSFE) is compared with alternatives approaches including the linear autoregressive model inferences. In this exercise, the moments and forecasting of the exchange rate are compared among models chosen by the Bayes factor (i.e., the LAR(1), the MSAR(2) and MSMAR(2) models in daily and monthly growth rates respectively). To do this, the optimal forecasting in MS frameworks will be shown to performs better than linear models. This, because the MSFE in-sample are lower in switching frameworks. Finally, in the out-of-sample exercise three periods ahead are inferred. In both frequency data, MS frameworks leads to lower forecasting error than linear inferences.

It is well known that the s-periods ahead forecast, converge to the sample mean in the autoregressive linear model, and to the unconditional mean in MS frameworks. However, in MS models the states of the time series are uncertain and they should be inferred as well. Therefore, in this paper this is estimated in three ways: first, conditional on the states which leads to K different paths; and second, forecasting by taking a weighted average of the conditional paths with weights given by the filtered probabilities at $T+1$, see Kim and Nelson (1999a) and Hamilton and Raj (2002). This approach is called the standard MS forecasting. The third approach is carried out according to Boot and Pick (2017).

They suggested and optimal forecasting approach in MS frameworks based on weights for the full sample. In general, the weights w are optimally chosen by minimizing the mean square forecast error (MSFE). Then the estimated filtered probabilities are replaced by general weights w_t for the forecast $\hat{y}_{T+1} = x'_{T+1}\hat{\beta}\,(w)$. This approach is called the optimal MS forecasting (Opt). That paper considered three different estimation of the weights based on the uncertainty about the hidden indicators of the states, but they demonstrated are asymptotically equivalent.

Therefore, to show that the in-sample inferences from MS frameworks perform better than linear modeling for both frequency exchange growth rate data, the Matlab code of Boot and Pick (2017) was applied to estimate the FSFE of the last 124 periods. In Table 8, the MSFE mean of the LAR, MS and optimal MS from three alternatives as mentioned before are reported. From this, in monthly data, all the means of the MSFE of MS modeling an optimal approaches are lower than the linear model (9.6974). On the other hand, in daily data, even though the difference are not so high like in monthly data. The mean of forecasting error from MS frameworks and two alternatives of the optimal weights (0.4292 and 0.4290), are also lower than the linear model (0.4333). Therefore, in both frequency exchange growth rates (Peso/Dollar) data, the in-sample forecasting mean for the last 124 periods under MS models beats the linear mean error. To save space any statistical test about these differences were omitted, but they can be carried out as in Boot and Pick paper.

Table 8. In-sample forecasting error

Frequency/Method	LAR(1)	MS(2)	Opt (1) MS	Opt (2) MS	Opt (3) MS
Monthly	9.6974	8.1926	8.1911	8.2625	8.2639
Daily	0.4333	0.4292	0.4292	0.4290	0.5166

Mean of the square forecast error (MSFE). The optimal forecasting is based on three estimation methods in Boot and Pick (2016)

In Table 9, the out-of-sample forecasting error of the exchange rates in levels at $T+1$ to $T+3$ are estimated to every model. As mentioned before, different models and methods are considered: the linear, LAR(1); applying the optimal weights in every state; the MS forecasting with two and three states; and inferences conditioning on every state. The square forecasting error (SFE) at $T+s$ with these approaches are reported as follows: from the MCMC output, the expected forecasting error i.e., $E\left[(x_{T+s} - \hat{x}_{T+s})^2\right]$ of the posterior mean in levels of the exchange rates were estimated. To save space the HDPI were omitted.

Table 9. Out-of-sample square forecasting error (SFE) at $T+s$

Date		Opt 1	Opt 2	Opt 3	LAR(1)	MS	MS(S_t=1)	MS(S_t=2)	MS(S_t=3)
29/05/2018	T+1	0.15	0.15	0.14	0.15	0.15	0.15	0.14	**0.07**
30/05/2018	T+2	0.12	0.12	0.11	0.13	0.12	0.14	0.12	**0.03**
31/05/2018	T+3	0.36	0.36	0.34	0.37	0.36	0.39	0.35	**0.15**
Mean		0.21	0.21	0.20	0.22	0.21	0.23	0.20	**0.08**
05/2018	T+1	1.17	1.18		1.16	1.18	1.20	**0.94**	
06/2018	T+2	2.00	1.98		1.94	1.97	2.01	**1.49**	
07/2018	T+3	0.82	0.77		0.68	0.75	0.80	**0.03**	
Mean		1.33	1.31		1.26	1.30	1.34	**0.82**	

The out-of-sample SFE was estimated as the square of difference from the estimated nominal exchange rate to the real data. Daily data (first rows) by the MSAR(2) model with three states, and monthly data (last rows) by the MSMAR(2) and two states. Only one lag was considered in linear model LAR. This based on the highest value of the ML in Table 3 and Table 4. The minimum FSE is marked in bold.

From this table, in daily data most of the MS methods beats the SFE of the linear model. The minimum SFE (marked in bold) are given by the MS forecasting error conditioning on the state three of high volatility. For the three periods ahead and the mean of the SFE this conditional forecasting is the minimum over all the methods and models. However, most of the optimal and MS forecasting errors are lower than the linear inferences, except the SFE of the state one with low volatility. In monthly data, only the SFE of the state of high volatility (marked in bold) is lower than the linear inferences in the three periods ahead. However, the linear model cannot be beaten by the optimal and the rest of MS frameworks in the out-of-sample forecasting error. The reason that, the optimal approaches and the rest of MS frameworks failed to provide lower out-of-sampling error might be explained by the low persistence of the states. From this exercise, there is always MS frameworks and methods that leads to lower forecasting errors in out-of-sample than the linear inferences at this particular sample period.

Finally in Table 10, the unconditional moments (mean and volatility) are reported for both frequency data and the MS modeling. This to compare the with the linear and the sample moments because the s-periods ahead converge to these moments. The unconditional moments in MS models are a weighted average of the state dependent conditional ones of Table 5, with weights given by the steady state probabilities in Table 6. As expected, the unconditional moments for the switching specifications are very close those of the LAR modeling and the true sample moments of the exchange growth rates. Furthermore, the three periods ahead exchange rates forecasting in levels (Peso/Dollar) are reported. That is, the LAR(1) model inferences against of those from the models chosen by the Bayes factor and methods that minimize the SFE of Table 9. From this table, it is important to note that the linear model sub-estimated the predicted level of the exchange rates in both frequency. This because, there is not any possible change of the state, different to the MS specification which considered this with some probability. In particular, in monthly data the linear inferences are lower and close to 19 pesos by dollar, and the real data is above of 19. In daily data, the linear inferences are also lower than the real data, but the MS inferences in the high volatility state are closer to the real exchange rates.

Table 10. Moments of the LAR and the unconditional moments of the switching modeling

Levels	**Monthly**			**Daily**		
Time	**Real**	**LAR(1)**	**Forecast(*)**	**Real**	**LAR(1)**	**Forecast(*)**
T+1	19.49	18.32	18.55	19.75	19.60	19.68
T+2	20.31	18.37	18.82	19.73	19.60	19.76
T+3	19.12	18.44	19.08	19.98	19.61	19.83
Growth rates	**Sample**	**LAR(1)**	**MSMAR(2)**	**Sample**	**LAR(1)**	**MSAR(2)**
Mean	0.44	0.41	0.29	0.02	0.02	0.02
Volatility	2.60	2.60	2.29	0.67	0.67	0.57

(*) means the forecasting level of exchange rate (Peso/Dollar) from the models and methods that minimize the FSE of Table 9.

In summary, under an appropriated model selection according to the best adjusting to the data, in-sample and out-of-sample forecasting of the Mexican exchange rate (Peso/-Dollar) in MS frameworks have a lower forecasting errors than those from linear modeling for this particular time period. In any case and frequency of the data, the MS frameworks cannot beaten the by the linear inferences.

5. Conclusions

This paper studied the econometric modeling and forecasting of the nominal Mexican exchange growth rates (Peso/Dollar) from 1995 to 2018. By applying Bayesian simulation methods based on Frühwirth-Schnatter (2004), there were found the best-fit to data econometric autoregressive models. Furthermore, an exercise of in-sample and out-of-sample forecasting among the linear and non-linear models by introducing MS parameters were carried out.

For two frequencies of the exchange growth rates data (daily and monthly), it was found evidence of changes in the mean and volatility of the market. A three states autoregressive models with state dependent volatility in high frequency of the data is strongly supported by the sample. There are some periods of high volatility with growth rate mean besides of those in the sub-prime crisis in 2008-2009. Furthermore, there is always evidence of changes between low, medium and high volatility. Therefore, a structural changes in linear models for the Mexican exchange rate are not empirically supported. On the other hand, in low frequency data (monthly exchange growth rates), different means and volatilities are also found. However, the persistence of each state is not high. That is, changes in the monthly volatility are more likely than the high frequency data of the exchange growth rates. However, in the last two years, the high volatility state is present with high probability.

In the forecasting exercise, the MS autoregressive models lead to that the forecasting errors of the exchange growth rates are lower than those of the linear modeling in both frequency data, which is an issue of strong discussion in most of the empirical literature. In financial emerging markets, it is expected strong financial shocks and markets are subject to different volatile periods (e.g., changes in the mean and the volatility of the exchange growth rate). Therefore, there is always a probability that the process changes to another state. To do this, the optimal weighting approach of Boot and Pick (2017) was considered in MS frameworks. Furthermore, it is well known that periods ahead of forecasting in linear models, converge to the unconditional mean of the process which might lead to higher errors than MS model inferences if changes of the states are omitted. The results an methods, might be very useful to decision makers since our prediction of changes in volatilities might help to inference the risk in this market.

Moreover, independent of the frequency data, the Bayes Factor provided enough evidence that any linear autoregressive model with constant parameters over time is a wrong specification to study the Mexican exchange rate. Moreover, imposing exogenous structural breaks might not be a reliable assumption since volatility changes are present in different time-periods of the sample beside of those from the sub-prime crisis. This paper suggests that the dynamics and inference of this market are better adjusted with any time-varying parameters econometric model.

On the other hand, small data problems are very usual in emerging markets (e.g., to estimate large number of parameters with small time series or few observations in some state). But, it is important to note that the Bayesian approach is not based on asymptotic statistical properties. Then, the number of observations might not be a serious problem for inference. Furthermore, the model selection can be always carried out solving the nuisance parameters problem in the literature. Then, simulation methods might perform better in inferences than standard methods of frequentist econometrics. From this, under time-varying parameters models, Bayesian econometrics is a very useful tool to make more accurate estimation and inference.

This paper contributes in the literature in the exchange market in two features: an appropriated useful econometric tool based on the dynamics of the data to study changes in the mean and volatility as measure of risk in the market at different data frequency, and an empirical evidence of forecasting in financial markets characterized by suddenly volatile changes.

Finally, this study attempts to motivate that future research about financial issues of explanation of shocks and changes in volatility, co-movements in economic variables, determinants and inference in the Mexican exchange market, should be based on models with non-constant parameters over time. Therefore, we should be careful with the Mexican exchange rate econometric modeling (co-integration, unit root test, VAR model, GARCH and structural models). This, because the economic implications might be state dependent, and a misspecified econometric model might lead to different inferences that support the conclusions. Finally, model comparison and inference under GARCH and ARFIMA models in MS frameworks are left for future research.

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